Mahalinga-Iyer, U. & Williams, D. J. (1985). *Géotechnique* **45**, No. 2, 317-320

TECHNICAL NOTE

Unsaturated strength behaviour of compacted lateritic soils

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KEYWORDS: compaction; laboratory tests, partial saturation; pavements and roads, residual soils; shear strength.

INTRODUCTION

In tropical and subtropical countries, surficial soils remain unsaturated for most of the time, and field moisture contents of road pavements usually lie below the optimum moisture content for compaction. In order to understand the field behaviour of these soils, study of their unsaturated shear strength is important.

In the past, many researchers have attempted to analyse the unsaturated behaviour of soils. Bishop (1959) proposed an effective stress equation for the partially saturated state using a factor χ, which depends on the degree of saturation of the soil. Fredlund, Morgenstern & Widger (1978) proposed a shear strength equation for unsaturated soils using two independent stress state variables, the net normal stress $(\sigma - u_a)$ and the matrix suction $(u_a - u_w)$

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \qquad (1)$$

where τ is the shear strength, c' is the effective cohesion, σ is the total normal stress, u_a is the pore air pressure, u_w is the porewater pressure, ϕ' is the angle of friction associated with $(\sigma - u_a)$ and ϕ^b is the angle of friction for changes in matrix suction.

Toll (1990) put forward a framework for unsaturated soil behaviour incorporating volume change as well. For the shear strength at the critical state he proposed the equation

$$q = M_a(p - u_a) + M_w(u_a - u_w) \qquad (2)$$

where p is the mean principal stress $(\sigma_1 + \sigma_2 + \sigma_3)/3$, q is the deviatoric stress $(\sigma_1 - \sigma_3)$, M_a is the total stress ratio and M_w is the suction ratio. The parameters M_a and M_w correspond to ϕ' and ϕ^b in equation (1).

This Technical Note presents the unsaturated shear strength parameters from these two theories for two different compacted lateritic soils. The parameters for Toll's theory were obtained using the peak, rather than the critical state, as the triaxial tests were carried out up to 10% axial strain and it was not possible to achieve critical state conditions within this strain level. Toll (1990) stated that lateritic soil continues to dilate and will not reach a true critical state. It was therefore appropriate to consider the peak values, at which M_a and M_w correspond to ϕ' and ϕ^b.

EXPERIMENTAL PROCEDURE

The testing was carried out on two lateritic soils developed from basalt and sandstone, respectively, found in south-east Queensland. These soils have been used for road pavements in south-east Queensland. The shear strength properties were studied on compacted specimens 100 mm in diameter and 200 mm high. The classification and compaction properties are given in Tables 1 and 2.

Triaxial specimens were prepared by compacting the soil using a standard compaction hammer (after curing at optimum moisture content (OMC) for 24 h) into a mould, in five approximately equal layers, with 25 blows for each layer. One set of specimens was sealed immediately after preparation and another two sets of specimens were dried back to 90% and 70% of OMC, respectively, before they were sealed. The specimens were cured for three weeks before testing. Single-stage, unconsolidated undrained triaxial tests were performed on each set of specimens at four different cell pressures (0 kPa, 20 kPa, 35 kPa and 70 kPa), at a strain rate of 1 mm/min. One multistage, saturated undrained triaxial test with pore pressure measurement was carried out at a strain rate of 0·045 mm/min on each lateritic soil type. Saturation before shearing was achieved by the application of a back-pressure of 290 kPa and by the flushing of de-aired water through the sample.

Suction measurement was carried out using the filter paper method (McQueen & Miller, 1968).

Manuscript received 31 March 1994; accepted 11 May 1994.
Discussion on this Technical Note closes 1 September 1995; for further details see p. ii.
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Suction specimens were obtained from the triaxial samples soon after they reached the peak stress and after shearing to a large strain beyond the peak, care being taken not to disturb the fabric.

RESULTS

Fredlund's theory

Equation (1) was proposed for unsaturated soils by Fredlund *et al.* (1978). This is an extension of the well-known Mohr–Coulomb equation for saturated soils. In three-dimensional space, this equation describes a planar surface tangential to the Mohr circles at failure. The friction angle ϕ^b, assumed to be constant, is the slope of the plot of τ against $(u_a - u_w)$, when $(\sigma - u_a)$ is held constant. However, Fredlund *et al.* (1987) showed that ϕ^b varies with suction and that the failure envelope is non-linear with respect to the matrix suction axis.

Table 3 gives the shear strength parameters at the saturated state for the compacted lateritic soils. Figs 1 and 2 show the τ and ϕ^b plotted against $(u_a - u_w)$ corresponding to a net normal stress at failure of 100 kPa. For the lateritic soil developed on basalt, the slope ϕ^b of the failure surface with respect to the matrix suction decreases sharply in the suction range 0–1000 kPa and then decreases slightly in the suction range 1000–8000 kPa. For the lateritic soil developed on sandstone, ϕ^b decreases sharply in the suction range 0–200 kPa and then decreases slightly in the range 200–1800 kPa. The curves of τ against $(u_a - u_w)$ show that the failure envelope with respect to the matrix suction axis is non-linear for both soils.

Toll's theory

Equation (2) was proposed for unsaturated soils by Toll (1990) after analysis of the behaviour of a lateritic gravel from Kenya. In the unsaturated state the deviatoric stress invariant q is coupled with total stress ratio M_a and suction ratio M_w. Both these ratios depend on the degree of saturation. As the degree of saturation reaches 100%, the two ratios approach the same value and the equation becomes

$$q = Mp' \tag{3}$$

Toll (1990) determined the values of M_a and M_w by grouping tests at similar degrees of saturation and applying the multiple regression technique. Initially, a trend of the variation of these parameters with degree of saturation was established and this was refined to give best-fit functions.

For the lateritic soils tested here, four sets of data were available at each degree of saturation.

Table 1. Properties of lateritic soil from basalt

Liquid limit	44·8%
Plasticity index	12·8%
Linear shrinkage	7·0%
Clay fraction	4·0%
Specific gravity	2·89
Optimum moisture content	25·50%
Maximum dry density:	1·55 Mg/m³

Sieve size: mm	19	9·5	6·7	2·36	1·18
Percentage passing	72	58	52	38	31
Sieve size: mm	0·60	0·3	0·15	0·075	
Percentage passing	27	25	19	16	

Table 2. Properties of lateritic soil from sandstone

Liquid limit	26·0%
Plasticity index	8·1%
Linear shrinkage	7·0%
Clay fraction	5·5%
Specific gravity	2·76
Optimum moisture content	11·0%
Maximum dry density:	2·02 Mg/m³

Sieve size: mm	19	9·5	6·7	2·36	1·18
Percentage passing	91	62	52	39	36
Sieve size: mm	0·60	0·3	0·15	0·075	
Percentage passing	32	28	28	28	

Table 3. Shear strength properties

Parent rock	c': kPa	ϕ'
Basalt	55	30·0°
Sandstone	70	27·5°

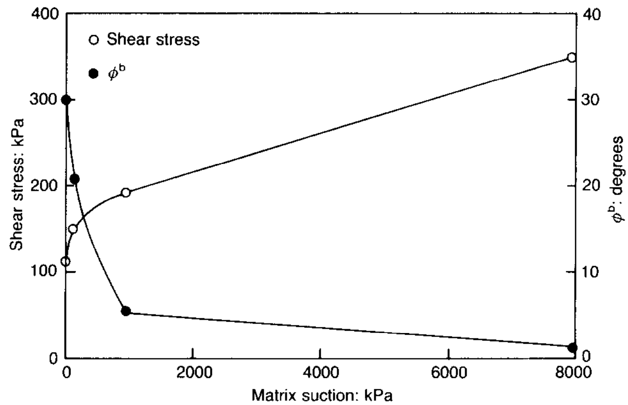

Fig. 1. Shear stress and ϕ^b plotted against matrix suction for lateritic soil from basalt

Using these four data points and applying the multiple regression technique, the values of M_a and M_w were determined at each degree of saturation. The coefficient of correlation r^2 was in the range 0·997–0·999.

Figures 3 and 4 show the variation of the stress ratios M_a and M_w with degree of saturation S for the lateritic soils tested, together with Toll's plots at the first stage of regression and at the refined stage of regression. For both lateritic soils the variation of M_a and M_w with S is similar to that obtained by Toll (1990). The values also lie in the same range. The suction has no effect on the deviator stress below a degree of saturation of 55% for both the soils tested. The corresponding limiting degree of saturation found by Toll was slightly lower at 53%.

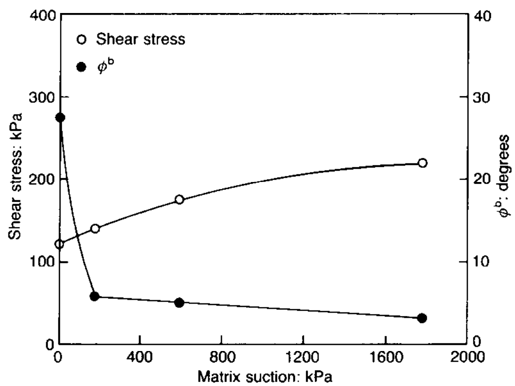

Fig. 2. Shear stress and ϕ^b plotted against suction for lateritic soil from sandstone

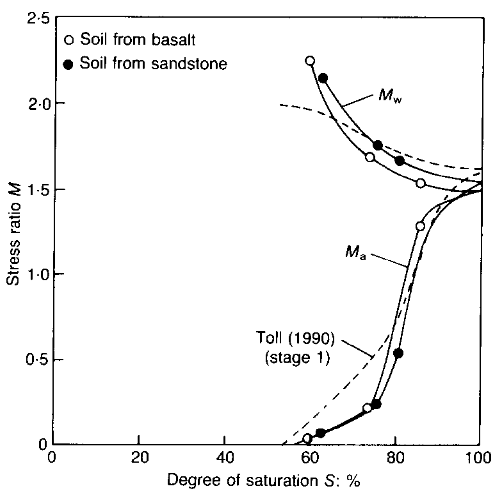

Fig. 3. Comparison of stress ratio–degree of saturation plots with Toll's (1990) stage 1 plot

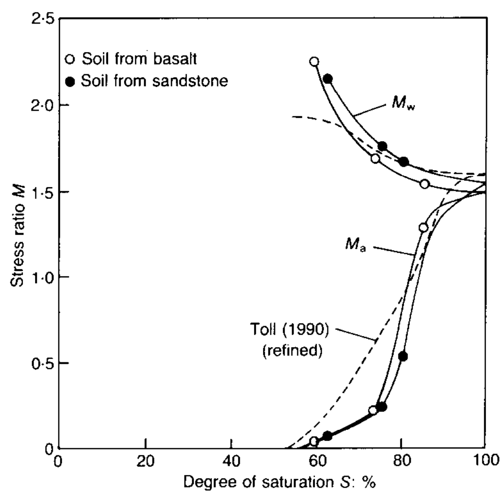

Fig. 4. Comparison of stress ratio–degree of saturation plots with Toll's (1990) refined plot

The suction value remained approximately constant after peak shear strength was reached in unsaturated, undrained triaxial testing. This indicates that there was no change in the fabric of the soil during the testing. The lateritic soil from basalt sustained higher values of suction than the lateritic soil from sandstone. This indicates that the soil from basalt has finer pores than that derived from sandstone.

DISCUSSION

To assess the in situ conditions of lateritic soils, the effect of soil suction should be included when the shear strength is evaluated. There is good agreement between the shear strength behaviour of the soils tested and Fredlund's theory. The results show a non-linear failure envelope in the plane of shear strength against matrix suction.

According to Toll's theory, there is a good relationship between the stress invariants and suction. The parameters determined from the current testing lie in the same range as those obtained from Toll's results on Kenya gravel. A similar limiting degree of saturation, beyond which suction has no effect, was obtained by Toll and in the current study. This limiting degree of saturation is in the range 53–55%.

CONCLUSIONS

Values obtained for Fredlund's parameter ϕ^b for the two lateritic soils tested vary with suction. Values obtained for Toll's parameters for the lateritic soils tested lie in the same range as his results for Kenya gravel. Suction has no effect on the shear strength of an unsaturated soil below a degree of saturation of 53–55%.

ACKNOWLEDGEMENTS

The Authors wish to express their gratitude to Dr P. H. Morris, Department of Civil Engineering, University of Queensland, for his helpful comments on the content of the Technical Note.

NOTATION

c' effective cohesion
M_a total stress ratio
M_w suction ratio
p mean principal stress
q deviator stress
S degree of saturation
u_a pore air pressure
u_w porewater pressure
κ factor related to suction
σ total normal stress
σ_1 major principal stress
σ_2 intermediate principal stress
σ_3 minor principal stress
τ shear stress
ϕ' effective friction angle
ϕ^b friction angle related to suction

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TECHNICAL NOTE

Stability of infinite slopes

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KEYWORDS: failure; friction; plasticity: shear strength; slopes.

INTRODUCTION

In a research programme on the stability of overtopping dykes in the Netherlands, attention is focused on the inner slope. Within the inner slope a parallel flow develops which affects the geomechanical stability. One of the aims of this research is to derive simple design rules, in particular for the inner slope gradient for this loading condition.

A one-dimensional approach establishes the traditional formula for the stability of infinite slopes (Taylor, 1948; Heafeli, 1948; Skempton & DeLory, 1957). The critical stress ratio of a slope determines the stability of material sliding downhill with a slip surface parallel to the ground surface (see Fig. 1). This idea incorporates ground-water flow. The critical angle for the slope, or angle of natural slope, follows from the ratio of the shear stress to the vertical stress.

The shear stress and normal effective stress components (where compression is taken as positive) within the soil layer at depth are determined from

$$\sigma_{nt} = \gamma h \sin \alpha \tag{1}$$

$$\sigma_{nn} = (\gamma - \gamma_w) h \cos \alpha \tag{2}$$

where α is the angle of the slope, γ is the unit weight of the soil, γ_w is the unit weight of water and h is the depth of the layer. The ratio of the shear stress and normal stress follows from

$$\left(\frac{\gamma}{\gamma - \gamma_w}\right) \tan \alpha = \frac{\sigma_{nt}}{\sigma_{nn}} \tag{3}$$

For the stress state on the parallel failure plane, the Coulomb yield criterion for a cohesionless material is

$$\sigma_{nt} = \tan \phi \sigma_{nn} \tag{4}$$

Manuscript received 7 February 1994; revised manuscript accepted 11 May 1994.
Discussion on this Technical Note closes 1 September 1995; for further details see p. ii.
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where ϕ is the friction angle. For a cohesionless material, the critical slope angle is hence given by

$$\left(\frac{\gamma}{\gamma - \gamma_w}\right) \tan \alpha = \tan \phi \tag{5}$$

This is a necessary condition for the stability of slopes, but is not sufficient. It is also necessary to prove that such a stress state can exist within the soil mass.

STRESS STATE IN A COHESIONLESS INFINITE SLOPE

The Mohr–Coulomb yield criterion describes the critical stress state within the soil layer

$$\tau_m = \sigma_m \sin \phi + c \cos \phi \tag{6}$$

where τ_m is the radius of the Mohr circle and σ_m is the centre of the Mohr circle. This criterion implies that, in addition to the stress components already mentioned, the normal stress parallel to the slope is important, because in the soil all stress tensor components need to be considered. The tangential stress along the slope σ_{tt} is related the normal stress σ_{nn}

$$\sigma_{tt} = C_0 \sigma_{nn} \tag{7}$$

where C_0 is the stress ratio between normal and tangential stress. The stress ratio C_0 is similar to K_0, but with respect to the normal and parallel axes of the slope (see Fig. 1). The stresses at failure that conform to the Mohr–Coulomb yield

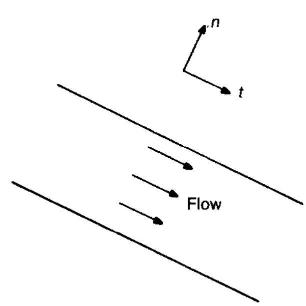

Fig. 1. Stability of single layer with parallel flow

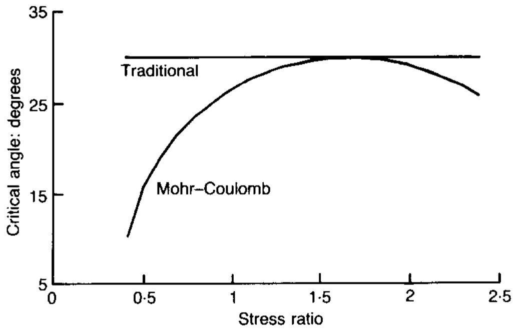

Fig. 2. Critical slope angles for a dry analysis: $\phi = 30°$

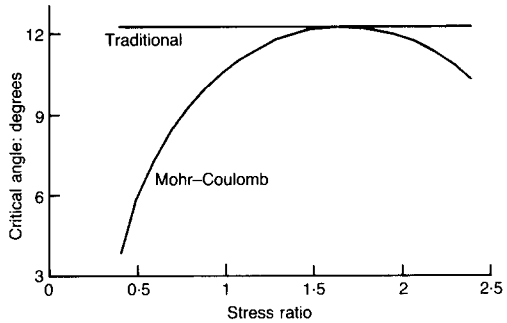

Fig. 3. Critical slope angles for parallel flow: $\gamma = 16$ kN/m^3, $\gamma_w = 10$ kN/m^3 and $\phi = 30°$

criterion are described by the expression

$$\left(\frac{\gamma}{\gamma - \gamma_w}\right) \tan \alpha = \sqrt{\left[(\sin \phi)^2 \left(\frac{1 + C_0}{2}\right)^2 - \left(\frac{1 - C_0}{2}\right)^2\right]} \quad (8)$$

which differs from the traditional formula for a wide range of C_0. Fig. 2 shows the result for a dry slope; Fig. 3 shows the result for a slope with parallel flow. The tangential stress σ_{tt} is important in addition to the stress ratio as described in equation (3).

The critical angles in expressions (5) and (8) become equal for the stress state

$$\sigma_{tt} = \frac{1 + (\sin \phi)^2}{1 - (\sin \phi)^2} \sigma_{nn} \quad (9)$$

The traditional formula for the critical slope is optimistic compared with the Mohr–Coulomb yield criterion. Only for a relatively high tangential normal stress σ_{tt} do equations (5) and (8) become identical. In this example for a friction angle of 30°, this means a stress ratio of 5/3, which is rather high. Moreover, these results indicate that the failure condition within the layer itself is more critical. No definite conclusions can be drawn from the evaluation of the stress state itself.

SIMPLE SHEAR MECHANISM IN AN INFINITE SLOPE

The plastic strain rates determine the value for the critical slope angle. Apart from the stress state, it is necessary that the plastic strains generate a kinematically admissible velocity field. The plastic potential describes the plastic strains and has the same form as the Mohr–Coulomb yield function (equation (6)), with the friction angle replaced by the dilatancy angle ψ

$$g = \tau_m - \sigma_m \sin \psi \quad (10)$$

Plane sliding parallel to the slope describes the failure mechanism. This situation is identical with simple shear. The condition for the velocity field in such a case is

$$\dot{\varepsilon}_{tt} = 0 \quad \text{or} \quad \partial g / \partial \sigma_{tt} = 0 \quad (11)$$

The condition that there be no tangential strain is the additional condition that determines the stress rate. From this condition, the expression for the slope angle follows

$$\left(\frac{\gamma}{\gamma - \gamma_w}\right) \tan \alpha = \frac{\sin \phi \cos \psi}{1 - \sin \phi \sin \psi} \quad (12)$$

This condition is identical, for the resulting co-axial model, with the limit stress ratio for simple shear conditions (Teunissen & Vermeer, 1988). Davis (1968) published the same result for velocity discontinuities based on the method of characteristics. The conditions for simple shear and for velocity discontinuities have a strong resemblance. Drescher & Detournay (1993) incorporate the results from Davis (1968) for limit load calculations for different materials.

For an associative material with identical dilatancy and friction angles, equation (12) reduces to the traditional formula (equation (5)). Equation (12) is more powerful because it is valid for a wider range of materials; it also shows that the dilatancy angle contributes to the limit value. If the dilatancy is zero, the tangent in equation (5) should be made a sine, resulting in a lower gradient. The range of critical slope angles becomes much wider by inclusion of non-coaxial plastic deformation terms (Teunissen & Vermeer, 1988).

The friction angle is a function of the dilatancy angle in equation (12). The stress–dilatancy relationship as formulated by Bolton (1986) decomposes the friction angle in terms of the friction angle for constant volume ϕ_{cv} and the dilat-